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Estimating DSGE Models under Partial Information *

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Abstract

Most DSGE models and methods make inappropriate asymmetric information assumptions. They assume that all economic agents have full access to measurement of all variables and past shocks, whereas the econometricians have no access to this. An alternative assumption is that there is symmetry, in that the information set available to both agents and econometricians is incomplete. The reality lies somewhere between the two, because agents are likely to be subject to idiosyncratic shocks which they can observe, but are unable to observe other agents' idiosyncratic shocks, as well as being unable to observe certain economy-wide shocks; however such assumptions generally lead to models that have no closed-form solution.

This research aims to compare the two alternatives - the asymmetric case, as commonly used in the literature, and the symmetric case, which uses the partial information solution of Pearlman *et al.* (1986) using standard EU datasets. We use Bayesian MCMC methods, with log-likelihoods accounting for partial information.

The work then extends the data to allow for a greater variety of measurements, and evaluates the effect on estimates, along the lines of work by Boivin and Giannoni (2005).

JEL Classification: C11, C13, D58, D82

Keywords: partial information, DSGE models, Bayesian maximum likelihood

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Contents

1 Introduction and Motivation

Central banks increasingly pay attention to a much larger number of time-series in their forecasts than is commonly assumed by academic econometricians. For example, the US Fed is thought to keep and evaluate thousands of series[1]. This has led to the recent developments in rich data MCMC and VAR estimation Boivin and Giannoni (2005), Jacquier *et al.* (2004) and Bernanke and Boivin (2003).[2] Many of the recent DSGE studies show significant superiority of DSGE over unrestricted VAR, structural S-VAR and Bayesian B-VAR forecasts, especially in the longer term (e.g. Smets and Wouters (2003), Smets and Wouters (2004), and Boivin and Giannoni (2005)).

However, most of this DSGE estimation makes asymmetric information assumptions where full information about past shocks is available to the economic agents but not to the econometricians. Although full information on idiosyncratic shocks may be available to economic agents, they are unlikely to have full information on economy-wide shocks. However, with agents unable to observe idiosyncratic shocks other than their own, this leads in general to models for which there is no aggregate closed-form representation. It therefore makes sense to address alternative information assumptions in order to assess whether parameter estimates are consistent across these assumptions. For the purposes of this paper we focus on a standard New Keynesian (NK) macromodel, and make the assumption that either agents are better informed than the econometricians (the standard asymmetric information case in the estimation literature) or that they both have only partial information available, and that there is informational symmetry. The reduced-form solution in the latter case was obtained for a completely general linear rational expectations model by Pearlman *et al.* (1986).

An additional attraction of working with the partial information setup is that the NK models have much improved properties in this case. Collard and Dellas (2004), Collard and Dellas (2004) have shown that with imperfect information about output and the technology shock, or with misperceived money, the effect on inflation and output of a monetary shock is the hump-shaped one displayed empirically. With full information, the hump-shaped effect is not in evidence in simulations of the NK model.

[1]Observers of Alan Greenspan's chairmanship, for example, have emphasized his own meticulous attention to a wide variety of data series Bernanke and Boivin (2003)

[2]For example, Bernanke and Boivin (2003) build upon the work of Stock and Watson (1999) who conclude that the best-performing forecast for inflation is an augmented Phillips curve forecast that uses a new composite index of aggregate activity comprised of the 168 individual activity measures.

2 Survey of Existing Literature and Previous Work

2.1 Bayesian MCMC DSGE

Forecasting of rational agents' behaviour has been seen as a step in resolving the Lucas Critique issues of dynamic changes of market agents' forward looking rational expectations and policy change Robert E. Lucas (1975).

Blanchard and Kahn (1980) provided a general solution for a linear model under RE in the state space form, the same year that Sims (1980) suggested use of Bayesian methods for solving multivariate systems which led to development of Bayesian VAR (BVAR) models Doan *et al.* (1984), and, during the 1980s, the extensive development and application of Kalman filtering-based state space systems methods in statistics and economics Aoki (1987), Harvey (1989).

Modern Dynamic Stochastic General Equilibrium (DSGE) methods further enhance this Kalman filtering based Bayesian VAR state space model with Monte-Carlo Markov Chain (MCMC) optimising, stochastic simulation and importance-sampling (Metropolis-Hastings or Gibbs) algorithms. The aim of this enhancement is to provide the optimised estimates of the expected values of the currently unobserved, or the expected future values of the variables and of the relational parameters together with their posterior probability density distributions Geweke (1999). It has been shown that DSGE estimates are generally superior, especially for the longer-term predictive estimation than the VAR or BVAR estimates Smets and Wouters (2003), Smets and Wouters (2004), and also in data-rich conditions Boivin and Giannoni (2005).

The crucial aspect is that agents in DSGE models are forward-looking. As a consequence, any expectations that are formed are dependent on the agents' information set. Thus unlike a backwards-looking engineering system, the information set available will affect the path of a DSGE system.

2.2 Partial Information Rational Expectations

Consider one of the popular models in DSGE estimation, that of Smets and Wouters (2003). This incorporates a variety of shocks, capital stock, and therefore Tobin's Q. The measurements that are used to estimate parameters never include the shocks and rarely the capital stock; if the latter is included, it will be regarded as a noisy measure of the true value. However the conventional approach in Dynare and in the Bayesdsge software of Justiniano assumes firstly that agents in the economy have all this information, but secondly that the econometricians do not. We refer to estimates arising from these assumptions as Asymmetric Estimates.

To illustrate the issue, consider the following, not completely general, setup. We assume that the dynamics of the system are given by

$$\begin{bmatrix} z_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_t \\ x_t \end{bmatrix} + \begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix} \tag{1}$$

where z_t, x_t are vectors of backward and forward-looking variables, respectively, and u_t is a shock variable; a more general setup allows for shocks to the equations involving expectations. In addition we assume that agents all make the same observations w_t at time t, which are given by

$$w_t = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} z_t \\ x_t \end{bmatrix} + v_t \tag{2}$$

where v_t represents measurement error. Given the fact that expectations of forward-looking variables depend on the information set, it is hardly surprising that the absence of full information will impact on the path of the system.

Following Blanchard and Kahn (1980), we know that, *assuming full information,* there is a saddle path satisfying:

$$x_t + N z_t = 0 \quad where \quad \begin{bmatrix} N & I \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \Lambda^U \begin{bmatrix} N & I \end{bmatrix} \tag{3}$$

where Λ^U has unstable eigenvalues. The overall system as estimated in Dynare is then given by:

$$z_{t+1} = (A_{11} - A_{12}N)z_t + u_{t+1} \qquad w_t = (K_1 - K_2 N)z_t + v_t \tag{4}$$

and the Kalman filter is implemented at this point in the procedure.

Before reviewing the correct reduced-form solution in the partial information case, we briefly examine the background to the partial information solution. Townsend (1983) showed that when there are agents with diverse (partial) information, the equilibrium dynamics of the system are more drawn out, and this is attractive for the purposes of matching data to theoretical models. His work was formalised by Pearlman and Sargent (2003), which was based on Pearlman *et al.* (1986). The latter (henceforth PCL), which assumes that all agents have the same (partial) information set, is the basis of our work.

The correct reduced-form solution, as derived by PCL, turns out to be the following:

$$System: \quad z_{t+1} = C z_t + (A - C)\tilde{z}_t + u_{t+1} \tag{5}$$

$$x_t = -N z_t + (N - A_{22}^{-1} A_{21})\tilde{z}_t \tag{6}$$

$$Innovations: \quad \tilde{z}_{t+1} = A\tilde{z}_t - APD^T(DPD^T + V)^{-1}(D\tilde{z}_t + v_t) + u_{t+1} \tag{7}$$

$$Measurement: \quad w_t = E z_t + (D - E)\tilde{z}_t + v_t \tag{8}$$

$$where \quad C = A_{11} - A_{12}N \quad A = A_{11} - A_{12}A_{22}^{-1}A_{21} \quad E = K_1 - K_2 N \quad D = K_1 - K_2 A_{22}^{-1} A_{21}$$

V is the covariance matrix of the measurement errors, and P is the solution of the Riccati equation given by

$$P = APA^T - APD^T(DPD^T + V)^{-1}DPA^T + U$$

and U is the covariance matrix of the shocks to the system. Clearly the correct dynamics of the system are more complex than is assumed within the solution procedure of Dynare.

The innovations process is defined as $\tilde{z}_t = z_t - E_{t-1}z_t$, so that it immediately obvious how to update estimates of the backward-looking variables. The likelihood calculations (see Appendix 1) then take account of this reduced form, which now assumes that agents and econometricians have the same information!

2.3 Timing Issues in Bayesdsge and Dynare

The most recent software for solving forward-looking RE models (including Dynare and Bayesdsge) applies a slightly different form of the model, and therefore its generalised solution. It locates z_t (rather than z_{t+1}) and $E_t x_{t+1}$ on the left hand side of the equation:

$$\begin{bmatrix} W & X \\ Y & Z \end{bmatrix} \begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u_t$$

with the information set selecting terms in z_t, x_t. In particular, this usually assumes no noisy measurements, but with only part of the state vector observed i.e.

$\mathrm{w_t} = \begin{bmatrix} \mathrm{L_1 z_t} \\ \mathrm{L_2 x_t} \end{bmatrix}$ where L_1, L_2 are selection matrices composed of 0s and 1s.

In order to match this to the results of PCL above, it is useful to reorganise the generalised model so that the shocks (typically AR1 processes) are included in the state vector z_t. This means that expectations $E_t x_{t+1}$ are only related to z_t, x_t with no shock terms; the latter only affect the dynamics of z_t. In effect what one does is first of all to invert the LHS matrix above to yield

$$\begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} \begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + \begin{bmatrix} \bar{H}_1 \\ \bar{H}_2 \end{bmatrix} u_t \tag{9}$$

and then to rewrite the system in a way that conforms to (1) and (2).

$$\begin{bmatrix} u_{t+1} \\ z_t \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \bar{H}_1 & \bar{A} & \bar{B} \\ \bar{H}_2 & \bar{C} & \bar{D} \end{bmatrix} \begin{bmatrix} u_t \\ z_{t-1} \\ x_t \end{bmatrix} + \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} u_{t+1} \tag{10}$$

$$w_t = \begin{bmatrix} L_1\bar{H}_1 & L_1\bar{A} & L_1\bar{B} \\ 0 & 0 & L_2 \end{bmatrix} \begin{bmatrix} u_t \\ z_{t-1} \\ x_t \end{bmatrix} \tag{11}$$

3 Data Rich Estimation and Forecasting

Instead of using one series as the indicator of a variable, Boivin and Giannoni (2005) (henceforth BG) list and provide comprehensive reasons for the use of multiple series. Firstly, is that adding data series correlated to the structural variables allows for (panel-data like) identification of patterns of measurement errors and subsequent reduction of the risk of biased estimation. The second advantage is that it provides a potential to yield a more efficient estimation procedure.

In their results they show that rich series provide more accurate estimations and forecasts than traditional DSGE estimations. However, their work is subject to the general criticisms on partial information that we have outlined above.

3.1 Implementation of PCL in the Data Rich Framework

BG utilize three essentially different types of measurements. The first just involves measurements of variables within the state vector, such as consumption or inflation; the second involves measurements of variables related to the state vector (such as net exports) but for which the linear relationship has to be estimated. Thirdly, either of the above types of measurement may be made with measurement error.

It is now straightforward to express this in the modified PCL format above, provided that one includes the measurement error shocks in the state vector. Note that if the measurement errors are serially uncorrelated, and one can include them <u>directly</u> as measurement errors in the vector v_t.

4 The Model

The model estimated is the single country version of the standard New-Keynesian model as defined in Smets and Wouters (2003) and in data-rich conditions in BG. The equations are all linearized about the steady state of the Ramsey optimum. The variables used, c, i, k, mc, wr, l, y, g represent proportional deviations from steady state of consumption, investment, capital stock, marginal cost, real wage, labour hours, output and government spending, while q, π, r, r_K, z represent deviations from steady state of Tobin's Q, inflation, interest rate, return on capital and capital utilization. The effect of fixed costs F in production is also incorporated, as in Smets and Wouters (2003).

$$
\begin{aligned}
c_t &= \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} \\
&- \frac{1-h}{(1+h)\sigma}(r_t - E_t\pi_{t+1} + E_t u_{C,t+1} - u_{C,t}) && (12) \\
q_t &= \beta(1-\delta)E_t q_{t+1} - (r_t - E_t\pi_{t+1}) + \beta E_t r_{K,t+1} + \epsilon_{Q,t} && (13) \\
z_t &= \frac{r_{K,t}}{Z\Psi''(Z)} = \frac{\psi}{R_K} r_{K,t} \quad \text{where } \psi = \frac{\Psi'(Z)}{Z\Psi''(Z)} && (14) \\
i_t &= \frac{1}{1+\beta}i_{t-1} + \frac{\beta}{1+\beta}E_t i_{t+1} + \frac{1}{S''(1)(1+\beta)}q_t + u_{I,t} && (15) \\
\pi_t &= \frac{\beta}{1+\beta\gamma_p}E_t\pi_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\pi_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p}mc_t + u_{P,t} && (16) \\
k_t &= (1-\delta)k_{t-1} + \delta i_{t-1} && (17)
\end{aligned}
$$

$$
\begin{aligned}
mc_t &= (1-\alpha)wr_t + \frac{\alpha}{R_K}r_{K,t} - a_t && (18) \\
wr_t &= \frac{\beta}{1+\beta}E_t wr_{t+1} + \frac{1}{1+\beta}wr_{t-1} + \frac{\beta}{1+\beta}E_t\pi_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} \\
&+ \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\xi_w(1+\eta\phi)}(mrs_t - wr_t) + \epsilon_{W,t} && (19) \\
mrs_t &= \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \phi l_t + u_{L,t} && (20) \\
l_t &= k_{t-1} + \frac{1}{R_K}(1+\psi)r_{K,t} - wr_t && (21) \\
y_t &= c_y c_t + g_y g_t + i_y i_t + k_y \psi r_{K,t} && (22) \\
y_t &= \phi_F[a_t + \alpha(\frac{\psi}{R_K}r_{K,t} + k_{t-1}) + (1-\alpha)l_t] \quad \text{where } \phi_F = 1 + \frac{F}{Y} && (23) \\
u_{C,t+1} &= \rho_C u_{C,t} + \epsilon_{C,t+1} && (24) \\
u_{L,t+1} &= \rho_L u_{L,t} + \epsilon_{L,t+1} && (25) \\
u_{I,t+1} &= \rho_I u_{I,t} + \epsilon_{I,t+1} && (26) \\
u_{P,t+1} &= \rho_P u_{P,t} + \epsilon_{P,t+1} && (27) \\
g_{t+1} &= \rho_g g_t + \epsilon_{g,t+1} && (28) \\
a_{t+1} &= \rho_a a_t + \epsilon_{a,t+1} && (29)
\end{aligned}
$$

where "inefficient cost-push" shocks $\epsilon_{Q,t+1}$, $u_{P,t+1}$ and $\epsilon_{W,t+1}$ have been added to value of capital, the marginal cost and marginal rate of substitution equations respectively.

Note that there are 5 forward-looking equations, but 6 forward-looking variables. One of these, $E_t r_{K,t+1}$, may be substituted by advancing all remaining equations by one period, taking expectations and solving in terms of all other variables.

We estimate an interest rate rule of the Taylor type, feeding back on current inflation deviations from target inflation $\bar{\pi}_t$ and on the output gap. The latter is the difference between deviations from trend output and deviations from the flexible-price natural rate $\hat{y}_t$.

$$r_t = \rho_r r_{t-1} + \rho_\pi(\pi_t - \bar{\pi}_t) + \rho_{\Delta\pi}\Delta(\pi_t - \bar{\pi}_t) + \rho_y(y_t - \hat{y}_t) + \rho_{\Delta y}\Delta(y_t - \hat{y}_t) + \varepsilon_{R,t} \tag{30}$$

In addition we assume that target inflation follows an AR(1) process:

$$\bar{\pi}_{t+1} = \rho_{\bar{\pi}}\bar{\pi}_t + \varepsilon_{\bar{\pi},t+1} \tag{31}$$

The natural rate is of course shock-dependent and not directly measurable. To model it, one assumes that prices and wages adjust immediately, and is subject to the same parameters as the main part of the model, which is otherwise unchanged. It thus derives from the flexi-price system given by:

$$\hat{q}_t = \beta(1-\delta)E_t\hat{q}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + \beta Z E_t\hat{r}_{K,t+1} \tag{32}$$

$$\hat{i}_t = \frac{1}{1+\beta}\hat{i}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{i}_{t+1} + \frac{1}{S''(1)(1+\beta)}\hat{q}_t + u_{I,t} \tag{33}$$

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{i}_{t-1} \tag{34}$$

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}E_t\hat{c}_{t+1} - \frac{1-h}{(1+h)\sigma}(\hat{r}_t - E_t\hat{\pi}_{t+1} + (\rho_C - 1)u_{C,t}) \tag{35}$$

$$0 = (1-\alpha)\hat{wr}_t + \frac{\alpha}{R_K}\hat{r}_{K,t} - a_t \tag{36}$$

$$\widehat{wr}_t = \frac{\sigma}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) + \phi\hat{l}_t + u_{L,t} \tag{37}$$

$$\hat{l}_t = \hat{k}_{t-1} + \frac{1}{R_K}(1+\psi)\hat{r}_{K,t} - \hat{wr}_t \tag{38}$$

$$\hat{y}_t = c_y\hat{c}_t + g_y g_t + i_y\hat{i}_t + k_y\psi\hat{r}_{K,t} \tag{39}$$

$$\hat{y}_t = \phi_F[a_t + \alpha(\frac{\psi}{R_K}\hat{r}_{K,t} + \hat{k}_{t-1}) + (1-\alpha)\hat{l}_t] \tag{40}$$

Note that in this flexi-price setup, only $\hat{i}_t, \hat{q}_t$ are forward-looking. Variables such as $E_t\hat{c}_{t+1}$ and $(\hat{r}_t - E_t\hat{\pi}_{t+1})$ depend directly on other current variables, and may in principle be solved for by advancing the system equations one period and taking expectations.

In addition, for the euro area there is no time series for hours worked available over the period 1970-2006 for which we have the rest of our data. Instead we use employment, and relate the two via a smoother which assumes that employment reacts more sluggishly in response to shocks than hours worked:

$$emp_t = \frac{1}{2}\left(emp_{t-1} + E_t emp_{t+1} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(l_t - emp_t)\right) \tag{41}$$

and we also estimate ξ_e.

As usual, some parameters are calibrated. Thus the discount factor is set at 0.99, and ratios of consumption, investment and government spending to income are averages over the time period. The parameter η, associated with labour's monopolistic power, is set to 3.

5 Data and Priors

5.1 Data

Although Boivin and Giannoni (2005) cover the period 1965-2002 and Smets and Wouters (2003) start back in 1957, all our estimation was performed on aggregated quarterly EU data covering the period 1970Q1 – 2005Q4 obtained from the Area Wide Model database.

The initial tests are performed primarily to identify the difference between estimation with asymmetric and (symmetric) partial information assumptions. The initial estimation is performed on the seven series: consumption, employment, output, inflation, investment, the interest rate and the real wage. The inflation rate is calculated as the first difference of the logs of the GDP deflator and all variables are detrended and de-meaned.

As an alternative to demeaning inflation and interest rate data, following Batini *et al.* (2005) we may use un-transformed inflation and interest rate data expressed in percent to estimate the mean, real inflation and unobserved real interest rates, and use the latter in the calculation of the (quarterly) future discount coefficient β as $\beta = 1/(1 + rr * /100)^{1/4}$ where rr^* is the unobserved, estimated real rate of interest in percentages. However this was not pursued here.

5.2 Priors

The priors for all parameters including standard deviations are taken from Choudhary *et al.* (2007), and are listed in the results section below.

Priors for standard deviations of the additional, indicator observations' relations to either a single or to multiple endogenous variables are square roots of the diagonal of covariance matrix of those series extracted using Eviews.

6 Estimation

6.1 Method and Tools

We used the BayesDSGE system of Matlab routines, a two stage solution variant of standard MCMC DSGE tool and based on the recursive State-space Kalman-Filter MLE (stage 1), and Bayesian MCMC Random-Walk Metropolis-Hastings algorithm (stage 2) methodologies. It was developed by Justiniano around Sim's gensys and csminwel programs, and used in Batini *et al.* (2005).

The system has been extended to the partial (symmetric) information Kalman filter based on the PCL solution, with the data rich generalisation. It also replaces Sim's gensys with the solution method outlined in the Timing Issues section above.

The project then

- Reproduces some of the results of DSGE asymmetric-information US macroeconomic parameter estimation and forecasting work by Boivin and Giannoni (2005) and Smets and Wouters (2003).
- Applies the enhanced, partial information method.
- Compares the estimates between both the asymmetric, and the new, partial, information assumption models, and the small and large number of measurement series, and outlines some recommendations for future enhancements.

We use four groups of estimates, along the lines of BG:

- **A** This set in part replicates case A of BG05 where only basic series are used. Our aim is to show the difference between the asymmetric and partial information estimation results using the three series y_t, i_t, π_t as explained in the Data section above. We assumed data to be free from measurement errors.

  (The following are still to be done):

- **B** This BG case extends case A to include possible measurement error.
- **C** As in BG, additional indicator series were added to the estimation. We use four additional observed indicator series, initially with an imposed restriction that observation series are functionally directly related to only one of the main model endogenous variables. We assume measurement error for each of the four:

  1. Inflation based on CPI, ($\pi_{cpi} = \rho_\pi \pi + \varepsilon_\pi$)
  2. deflated and detrended personal consumption ($c_p = \rho_y y + \varepsilon_y$)
  3. deflated and detrended hourly real wage ($w_h = \rho_w wr + \varepsilon_w$),
  4. detrended unemployment rate u, ($u = -\rho_u y + \varepsilon_u$)

- **D** In this case, the (strictly limited) indicator observation series are not restricted to a single system variable but may be related to several or all of them. As in C, all of these relationships with the system variables have to be estimated.

7 Results

The table below presents the preliminary results of our estimation. It shows the results from the full (asymmetric) information and the partial (symmetric) information cases. The latter so far covers only the case using the basic series, namely case A. In the middle columns are the means and standard deviations of the priors, with their densities N (normal), B (beta), I (inverse gamma), the latter with degrees of freedom rather standard deviation.

We present results for both maximum likelihood estimation (MLE) and for the medians of the second stage of the MCMC calculations. The MCMC is initialized in each case by the corresponding MLE. The first stage used 15,000 draws, then a burn-in of 1,000 draws, followed by another 15,000 draws.

For the full information case, the results are slightly puzzling, in that from past experience, on smaller models, the median estimates and MLEs tend to be fairly similar. This is not the case here, most notably for the parameter ξ_p, where $1/(1-\xi_p)$ represents the average number of quarters between optimal price-setting. The MLE of this is only .001 away from the median estimate obtained by Choudhary et al (2007) using Dynare, but our median estimate is 0.654. This would be very satisfying, as this is the most troubling parameter in DSGE estimation, but does not match other estimates from the literature using the same model. All other parameters match those found elsewhere.

Results for the partial information case are extremely encouraging, as the match between median estimates and MLEs is close. Even better is the estimate of ξ_p, which yields an average time between new price-setting of 4 quarters. Consumption habit h_C is a little on the high side, 0.7 as against 0.6, and the standard error on consumption preferences σ_c is high as well. This is perhaps an indication of the fact that the partial information approach, although attractive, is not ideal. It assumes that none of the shocks are directly observed by any of the agents in the economy, whereas the full information approach assumes that they all are. Since shocks to consumer preferences stem from individuals themselves, it is possible to assume that these are directly observable. However it is not clear how to model this intermediate situation.

8 Conclusions

We have presented the theoretical background to estimating a forward-looking expectations model under partial information, and used it to estimate a DSGE model for the euro area.

The main observations are that the parameter estimates under full and partial information are broadly similar. The highlight of the partial information estimates is that the average length of time between price-setting matches the microeconomic measure.

The main conclusion to be drawn this study is that for estimates to be reliable, one should perform estimation under both asymmetric and symmetric information assumptions. On the theoretical front, our results also highlight the need for a method of estimation that allows for the asymmetry of information about some shocks but not about others.

	Full	Information	Prior			Partial	Information
Parameter	Median	MLE	Density	Mean	SD	Median	MLE
$S''(1)$	6.365	6.241	N	4	1.5	5.693	5.868
σ	1.894	1.850	N	1	0.375	1.924	1.875
h_C	0.606	0.575	B	0.5	0.2	0.692	0.702
ϕ	2.486	1.884	N	2	0.75	1.220	1.133
ϕ_F	1.378	1.437	N	1.45	0.125	1.369	1.354
ψ	2.152	1.969	N	1	0.5	2.004	1.983
ξ_w	0.679	0.798	B	0.75	0.05	0.887	0.893
ξ_p	0.654	0.893	B	0.75	0.05	0.728	0.740
ξ_e	0.767	0.804	B	0.5	0.15	0.741	0.745
γ_w	0.708	0.498	B	0.5	0.15	0.587	0.623
γ_p	0.087	0.120	B	0.5	0.15	0.100	0.087
ρ_π	1.722	1.671	N	1.7	0.1	1.643	1.644
$\rho_{\Delta\pi}$	0.268	0.222	N	0.3	0.1	0.142	0.137
ρ_r	0.879	0.871	B	0.8	0.1	0.898	0.902
ρ_y	0.140	0.200	N	0.125	0.05	0.216	0.215
$\rho_{\Delta y}$	0.010	0.004	N	0.00625	0.05	0.022	0.020
ρ_a	0.990	0.990	B	0.85	0.1	0.990	0.990
ρ_c	0.925	0.953	B	0.85	0.1	0.959	0.958
ρ_g	0.930	0.927	B	0.85	0.1	0.891	0.893
ρ_L	0.974	0.980	B	0.85	0.1	0.819	0.853
ρ_I	0.947	0.951	B	0.85	0.1	0.950	0.954
$\rho_{\bar{\pi}}$	0.865	0.922	B	0.85	0.1	0.882	0.921
ρ_P	0.943	0.308	B	0.5	0.15	0.922	0.925
σ_a	0.601	0.701	I	0.4	2	0.729	0.762
σ_c	1.277	1.231	I	1.33	2	3.396	3.281
σ_g	1.641	1.620	I	1.67	2	1.582	1.569
σ_L	3.169	3.320	I	1	2	0.596	0.457
σ_I	0.057	0.047	I	0.1	2	0.082	0.076
$\sigma_{\bar{\pi}}$	0.013	0.009	I	0.02	10	0.011	0.009
σ_P	0.114	0.182	I	0.15	2	0.085	0.077
σ_r	0.574	0.565	I	0.1	2	0.528	0.531
σ_Q	0.529	0.524	I	3.2	2	0.501	0.491
σ_W	0.159	0.157	I	0.25	2	0.149	0.146

Appendix 1. The Filtering and Likelihood calculations The Kalman filtering equation is given by

$$z_{t+1,t} = Cz_{t,t-1} + CP_tD^T(EP_tD^T + V)^{-1}e_t \tag{42}$$

where $e_t = w_t - Ez_{t,t-1}$

$$P_{t+1} = AP_tA^T + U - AP_tD^T(DP_tD^T + V)^{-1}DP_tA^T \tag{43}$$

the latter being a time-dependent Ricatti equation.

The period-t likelihood function is standard:

$$2lnL = -\sum lndet(cov(e_t) - \sum e_t^T(cov(e_t))^{-1}e_t \tag{44}$$

where

$$cov(e_t) = (EP_tD^T + V)(DP_tD^T + V)^{-1}(DP_tE^T + V) \tag{45}$$

Following PCL, the system is initialised at

$$z_{1,0} = 0 \qquad P_1 = P + M \tag{46}$$

where $P$is the steady state of the Riccati equation above, and M is the solution of the Lyapunov equation

$$M = CMC^T + CPD^T(DPD^T + V)^{-1}DPC^T \tag{47}$$

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